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Chapter One
Charge, Electric field & Gauss's Law

1. What is Electrostatic induction?

Electrostatic induction is a method to create or generate static electricity in a material by bringing an electrically charged object near it. This causes the electrical charges to be redistributed in the material, resulting in one side having an excess of either positive (+) or negative (−) charges.

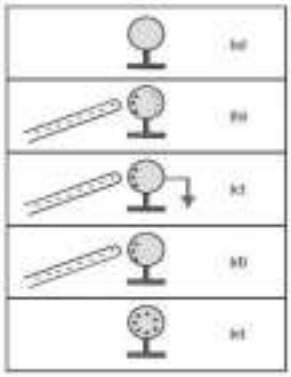


Figure 1 Electrostatic induction (Positive Charge)

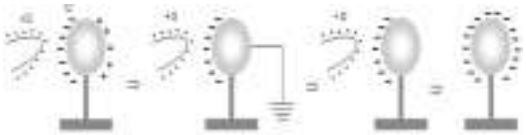


Figure 2 Electrostatic induction (Negative Charge)

2. Define: Charge, electricity.

Charge:
The fundamental and characteristics property of the particles (electron, proton etc.) that constitute matter is called charge. It is the property of material due to which, it exert a force. **Electricity:**
The nature and effect of static or moving charge is called electricity.

3. What is meant by Quantization of charge?

The charge of an electron or a proton is the minimum charge available in nature. Its magnitude is 1.60218×10^{-19} c. All other charges are the multiple of this smallest charge. It is called the quantization of charges. In nature, there is no existence of friction of the value of e. For example, 2.5e or -7.1e are not available in nature. Thus, If quantization is Q, we can write

$$Q = ne, \text{ where, } n = 0, 1, 2, 3\ldots\ldots$$

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4. What is electric field?

Electric Field:

The region around the electric charge in which the stress or electric force act is called an electric field or electrostatic field. If the magnitude of charge is large, then it may create a huge stress around the region. The electric field is represented by the symbol E. The SI unit of the electric field is newton per coulomb which is equal to volts per meter.

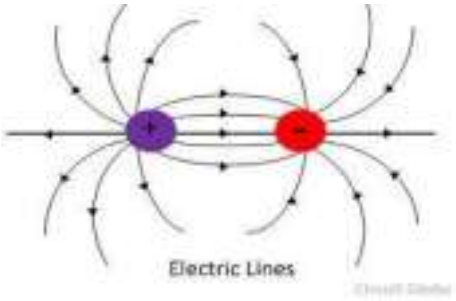


Figure 4

5. What is meant by electric field intensity?

Electric field strength or intensity:

The electric force acting on a unit charge at a point in the electric field is called electric field strength or electric field intensity.

It is denoted by E. It is a vector quantity. It can be written as, $E = F/q$

Vector form,

$$\vec{E} = \vec{F}/q$$

Where, F is the force acting on a point charge q.

The S.I. unit of electric field strength is N C^{-1}.

6. Write the expression for electric field intensity due to point charge?

The electric field due to a point charge:

Let a test charge is q placed at a distance r from a point charge q_0. The magnitude of force acting on q_0 is given by

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$$



Figure 5

Hence the electric field strength at point p is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^3}\vec{r}$$

$$E = \frac{q_0}{4\pi\epsilon_0}$$

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7. What is electric dipole?

An electric dipole is a separation of positive and negative charges. If a pair of electric charges of equal magnitude but opposite sign, separated by some distance, then the system is called an electric dipole.

8. What is meant by electric dipole moment?

The electric dipole moment is a measure of the separation of positive and negative electrical charges within a system, that is, a measure of the system's overall polarity. The SI units for electric dipole moment are coulomb-meter (C·m)

9. Calculate electric field intensity (E) for an electric dipole.

Electric Field Due to a dipole:

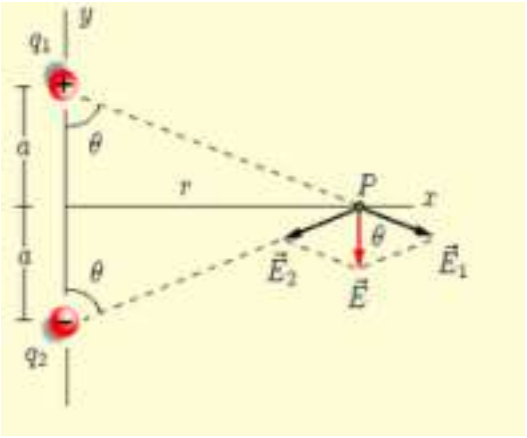


Figure shows an electric dipole consisting of two equal but opposite charges +q and −q which are at 2a distance apart. Now we find the expression for electric field at a point p, a distance r from the dipole. Let E_1 and E_2 be the electric field due to charge +q and −q respectively. Now, total electric field

$$\vec{E} = E_1 + E_2$$

Where $E_1 = E_1 = \frac{q}{(\sqrt{rr^2+aa^2})^2} = \frac{1}{4\pi\epsilon_0}$

Figure 6 $= \frac{1}{4\pi\epsilon_0}$ and E_2 $\frac{q}{rr^2+aa^2}$

The vector sum $\vec{E}$ of E_1 points vertically downward and has the magnitude. $E = \sqrt{aa_1^2 + aa_2^2 + 2aa_1aa_2\cos 2\theta}$

$$E = \sqrt{aa_1^2 + aa_1^2 + 2aa_1aa_1\cos 2\theta} \quad [\because E_1 = E_2]$$

$$E = \sqrt{2aa_1^2 + 2aa_1^2\cos 2\theta}$$

$$= \sqrt{2aa_1^2(1 + \cos 2\theta)}$$

$= \sqrt{2}\,\text{[illegible]}_1^2 2\,\text{[illegible]}^2\,\text{[illegible]}$

$\sqrt{4}\,\text{[illegible]}_1^2\,\text{[illegible]}^2\,\text{[illegible]}$

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$$E = 2E_1 \cos\theta$$

From figure, $\cos\,\text{[illegible]} = \frac{\text{[illegible]}}{\sqrt{\text{[illegible]}^2+\text{[illegible]}^2}}$

$$\therefore E = 2E_1 \cos\theta$$

$$= 2\,\frac{1}{4\,\text{[illegible]}\,\epsilon_0}\,\frac{\text{[illegible]}^2+\text{[illegible]}}{\sqrt{\text{[illegible]}^2+\text{[illegible]}^2}}\,\frac{\text{[illegible]}}{2\,\text{[illegible]}}$$

$$= \frac{1}{4\,\text{[illegible]}\,\epsilon_0}\,\frac{q}{(\text{[illegible]}^2+\text{[illegible]}^2)^{3/2}}$$

we get

But if we assume that $\text{[illegible]} \gg \text{[illegible]}$

$$E = \frac{1}{4\,\text{[illegible]}\,\epsilon_0}\,\frac{(2\,\text{[illegible]})\,\text{[illegible]}}{\text{[illegible]}^3}$$

Again the product (2a)q is called the electric dipole moment p.

$$E = \frac{1}{4\,\text{[illegible]}\,\epsilon_0}\,\frac{\text{[illegible]}}{\text{[illegible]}^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Equation (1) represents the electric field due to a point charge.

10. What is meant by charge density?

The charge density of a surface of any substance is defined as the total amount of charge existing around a point per unit area. Sometimes it is called surface charge density. Let

Q = amount of charge

A= area of cross section of substance

Charge density, $\text{[illegible]} = \frac{\text{[illegible]}}{\text{[illegible]}}$

Unit: Cm^{-2}

11. Write the expression of electric field intensity due to a ring of charge q and of radius a.

Electric field due to a ring of charge q and of radius a:

Let us consider a small amount of charge dq in the ring. The total electric field due to a points on the ring is Ex.

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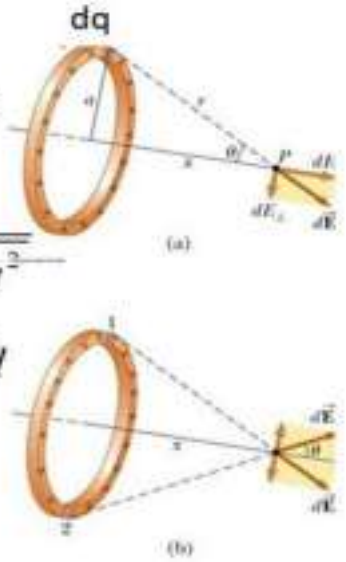
So,

$$dE_x = k_e \frac{dq}{r^2}\cos\theta = k_e \frac{dq}{x^2+a^2}\cos\theta$$

$$= k_e \frac{dq}{x^2+a^2}\frac{x}{r} = k_e \frac{dq}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}}$$

$$E_x = \int \frac{k_e x\,dq}{(x^2+a^2)^{3/2}} = \frac{k_e x}{(x^2+a^2)^{3/2}}\int dq$$

$$= \frac{k_e xQ}{(x^2+a^2)^{3/2}}$$



At altitude $\text{}_1 = 60$ km
E = 100 N

So,

If we assume that $\gg$ we get

$$E = E_x = \frac{}{^3} = \frac{1}{4} \frac{}{} = \frac{}{4 \in_0}$$

$$\therefore E = \frac{1}{^2}$$

12. **Deduce an expression of electric field intensity due to a long straight, uniformly charged wire. Or, Figure shows a section of an infinite line of a charge whose linear charge density (that is, the charge per unit length, measured in coul /meter) has the constant value . Calculate the field E a distance y from the line.**

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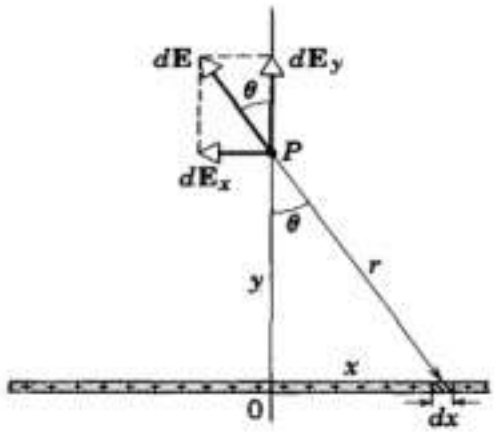


Figure 8. A section of an infinite line of charge

The figure shown a section of an infinite line of charge whose linear charge density (that is, the charge per unit length, measured in coul/meter) has the constant value λ.

The magnitude of the field contribution dE due to charge element dq = λdx is given by

$$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{y^2 + ••^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

According to the figure we can see that dE sin•• must be zero because every charge on the right has a corresponding element on the left such that their field contributions in the x direction cancel.

$$E = \int •••• = \int_{-••}^{••} ••••••••• ••••$$

$$= \int_{0}^{••} 2•••••••••••••$$

$$= 2\int_{0}^{••} \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{y^2 + ••••^{2}_{•••}}$$

According to the geometry rules we can write, ••

$$E = 2\int_{0}^{••} \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{y^2 + ••^2}\frac{y}{\sqrt{y^2 + ••^2}}$$

$$= 2\int_{0}^{••} \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{y^2 + ••^2}\frac{y}{(y^2 + ••^2)^{1}}$$

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$$= 2\int_0^{\infty} \frac{1}{4\pi\epsilon_0} \frac{y\lambda dx}{(y^2 + x^2)^{3/2}}$$

$$= \lambda \int_0^{\infty} \frac{2\pi\epsilon_0 y dx}{(y^2 + x^2)^3}$$

Again we know, $x = x\cos\theta$ and $x = x\tan\theta$

$\therefore dx = x\sec^2\theta\, d\theta$ [by differentiating]

When $x = 0$, then $\theta = 0$

When $x = \infty$, then $\theta = \frac{\pi}{2}$

$$\therefore E = \frac{\lambda}{2\pi\epsilon_0} \int_0^{\frac{\pi}{2}} \frac{x^2 x^2 \sec^2\theta\, d\theta}{(y^2 + x^2 \tan^2\theta)^{3/2}}$$

$$E = \frac{\lambda}{2\pi\epsilon_0} \int_0^{\frac{\pi}{2}} \frac{x^2 x^2 \sec^2\theta\, d\theta}{\{y^2(1 + \tan^2\theta)\}^{3/2}}$$

$$= \frac{\lambda}{2\pi\epsilon_0} \int_0^{\frac{\pi}{2}} \frac{x^2 x^2 \sec^2\theta\, d\theta}{(y^2)^{3/2} (\sec^2\theta)^{3/2}}$$

$$= \frac{\lambda}{2\pi\epsilon_0} \int_0^{\frac{\pi}{2}} \frac{x^2 x^3 \cos\theta\, d\theta}{y^3}$$

$$= \frac{\lambda}{2\pi\epsilon_0} \int_0^{\frac{\pi}{2}} \cos\theta\, d\theta$$

$$= \frac{\lambda}{2\pi \in_0 ••} \int_0^{••} \cos\theta \, d\theta$$

$$= \frac{\lambda}{2\pi \in_0 ••} [\sin\theta]_0^{••}$$

$$= \frac{\lambda}{2\pi \in_0 ••}$$

Which is the required expression for electric field intensity at any point due to a line of charge.

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13. Expression for Torque and potential energy of an electric dipole in a uniform electric field. Or, Derive the Equation of Torque and stored energy when an electric dipole is placed in a uniform electric field.

Figure shows an electric dipole consisting of two equal but opposite charges +q and −q that are placed a distance 2a apart. This arrangement is placed in a uniform electric field $\vec{E}$. The dipole moment p(=2aq) makes an angle θ with $\vec{E}$. The two forces $+\vec{F}$ and $-\vec{F}$ act as shown in figure where

$$F = qE$$

So, net force is zero but there is a net torque about an axis through O which is given by

$$\tau = 2F(a\sin\theta)$$

$$\tau = 2aF\sin\theta$$

$$= 2aqE\sin\theta$$

$$= PE\sin\theta$$

$$\therefore \vec{\tau} = \vec{P} \times \vec{E}$$

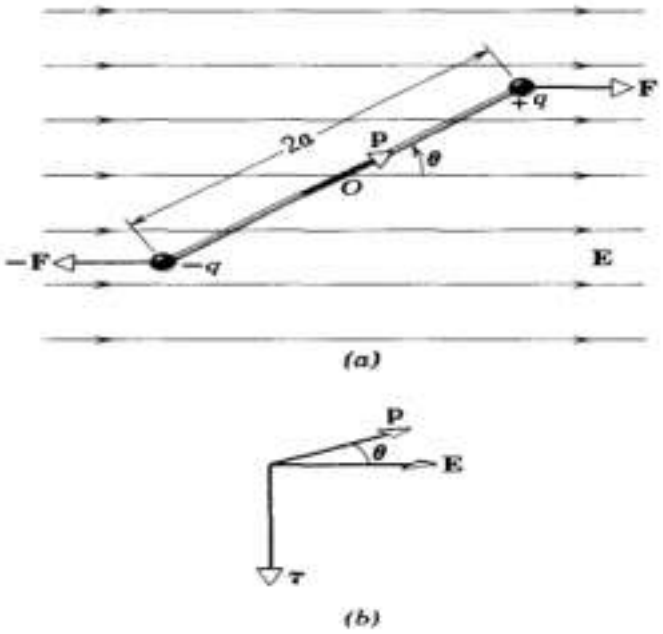


Figure 9 (a) An electric dipole in a uniform external field (b) An oblique view, illustrating

$$\vec{\tau} = \vec{P} \times \vec{E}$$

Now work must be done to change the orientation of an electric dipole by an external agent and this work is stored as potential energy.

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If θ has an initial value of θ_0, The work require to change the dipole axis to an angle θ is given by

$$W = \int dw = \int_{\theta_0}^{\theta} \tau \, d\theta = U$$

$$\theta = \int PE \sin\theta \, d\theta$$

$$\theta_0$$

$$= PE \left[-\cos\theta\right]_{\theta_0}^{\theta}$$

$$= PE(-\cos\theta + \cos\theta_0)$$

If we choose reference orientation θ_0 to have any convenient value $\theta_0 = 90°$, , we have $U = -PE\cos\theta$

$$\therefore U = -\vec{P} \cdot \vec{E}$$

Which is the required expression for potential energy of a dipole in an uniform electric field.

14. What is line of forces?

Line of forces: If a unit positive charge is placed in an electric field then the path through the charge moves are called electric lines of forces of that electric field. **15. Properties of electric lines of force.**

Properties of electric lines of force:

Electric lines of force starts from positive charge and end at the negative charge.
Electric lines of force originates and finishes pre-perpendicularity to the surface.
Electric lines of force never intersect each other.
Electric lines of forces repulses one another.
Electric lines of forces do not pass through a conductor.
When the charges are positive the electric field lines are down outward from the charges, when the charges are negative the electric field lines are down inward from the charge, when both positive and negative charges are present in the electric field, electric field lines generated from the positive charge and terminated at the negative charges. When both charges are same the mutual electric field lines are as shown in figure.

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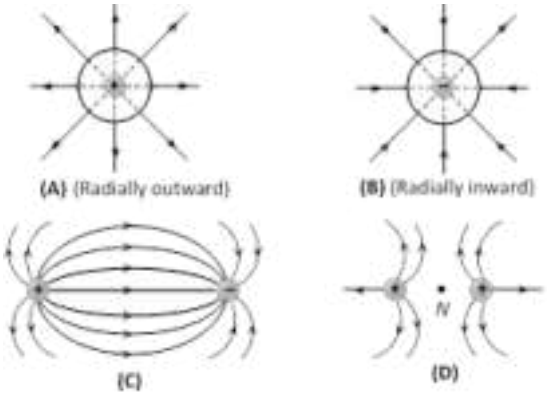


Figure 10. Electric lines of force

16. Define: Electric Flux, Electric Potential.

Electric flux: The number of electric lines of force or field lines passing through a pale or surface is called electric flux. It is denoted by Φ_E. Flux is property of any vector field. It refers to a hypothetical surface in the field, which may be open or closed. For closed surface, Φ_E is positive if the lines of force point outward and it is negative for that of inward.

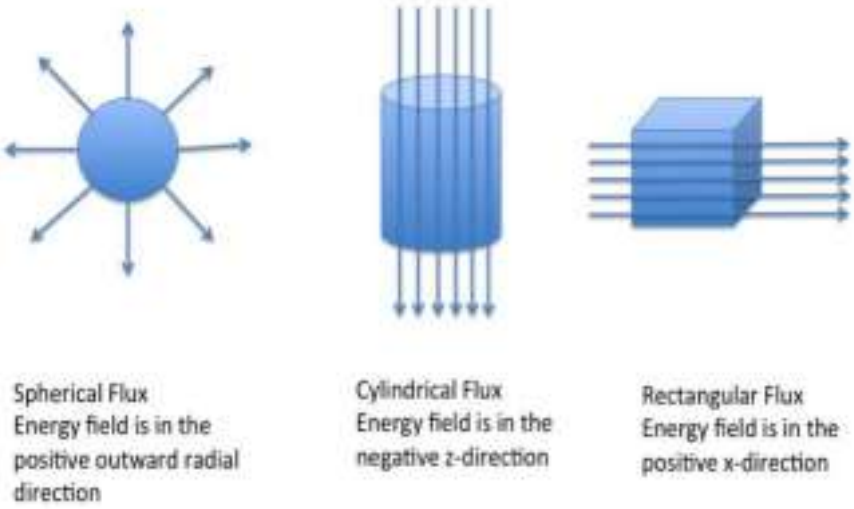


Figure 11 Electric flux

Electric potential:
The energy required to bring unit charge from infinity to a point in the electric field is known as electric potential.

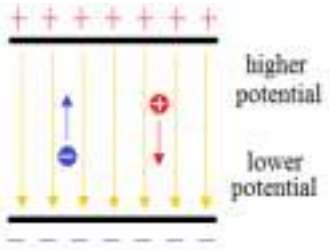


Figure 12

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17. State and explain Coulomb's law.

Coulomb's Law:
In 1785 Charles Augustin de Coulomb first measured electrical attractions and repulsions quantitavely and deduced a law that governs them which is known as coulomb's law. The law can be stated as below:
"The attractive or repulsive force between two charges is directly proportional to the product of two charges and inversely proportional to the square of their distance and the force acts along their joining line.
Explanation:
Let q_1 and q_2 be two point charges and the distance between them is r. If they assert a force of magnitude F, then according to Coulomb's law,

$F \propto q_1 q_2$ When, r is constant and

$F \propto \frac{1}{r^2}$ When q_1 and q_2 are constant

When r, q_1 and q_2 all vary, then

In vector form, $F \propto \frac{q_1 q_2}{r^2}$

or, $F = k\frac{q_1 q_2}{r^2}$

$\vec{F} = kq_1q_2\frac{\hat{r}}{r^2}$

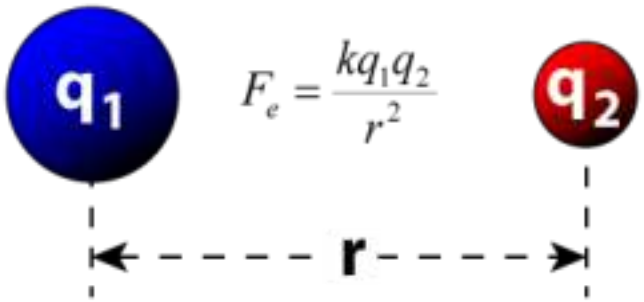


Figure 3

Here, k is a proportionality constant. Its value depends on the nature of the medium and the units of q_1, q_2, F and r.
In S. I. units,

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{nm}^2\text{c}^{-2}$$

The quantity ϵ_0 is called the permittivity constant**,** sometimes appears separately in equations and is

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$$\epsilon_0 = 8.854 \times 10^{-12}\ \text{N}^2/\text{Nm}^2$$

18. State and explain Gauss's law.

Gauss's Law
Eminent mathematician Karl F. Gauss gave this law.
The total charge enclosed by closed surface is equal to ϵ_0 times the electric flux with the closed surface.
If the area of a closed surface is S and the total charge enclosed by that surface is q, then according to Gauss's law,

$$\epsilon_0 \Phi = q$$

$$\text{Or, } \epsilon_0 \oint \vec{E} \cdot \vec{dA} = q$$

Here, ϵ_0 is the permittivity of vacuum.

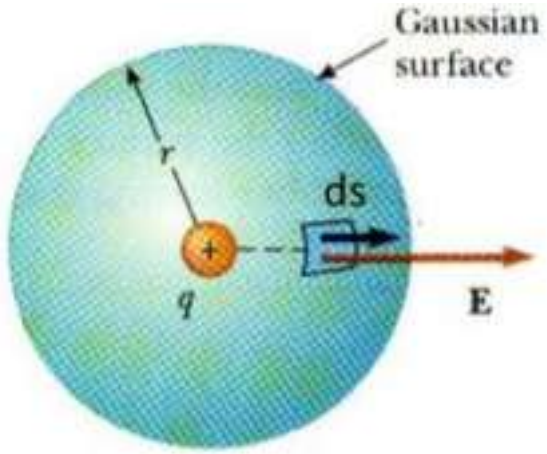
19. What is meant by Gaussian surface?
Gaussian surface
The imaginary closed surface having a collection of positive and negative charges which is used to explain the |Gauss's law is called the Gaussian surface.

20. Deduce Coulomb's law from Gauss's law.

Coulomb's law from Gauss's law:

Let us consider an isolated point charge q surrounded by a spherical Gaussian surface of radius r.

Now, $\vec{E}$ must be normal to the surface and must have some magnitude for all points on the surface.



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Figure 13

According to Gauss's law,

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

Since, $\vec{E}$ and $d\vec{A}$ at any point on the Gaussian surface are directed outward, so the angle between them is Zero.

$$\therefore \epsilon_0 \oint E \cdot dA \cos\theta = q$$

$$\text{বা, } \epsilon_0 \oint E \cdot dA \cos 0° = q$$

$$\text{বা, } \epsilon_0 \oint E \cdot dA = q$$

$$\text{বা, } \epsilon_0 E \oint dA = q$$

$$\text{বা, } \epsilon_0 E (4\pi r^2) = q$$

Where, $\oint dA = 4\pi r^2$ is the area of the surface.

$$E = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ldots\ldots\ldots (1)$$

Equation (1) gives the magnitude of $\vec{E}$ at any point a distance r from an isolated point charge q.

Let us put a second point charge q_o at the point at where $\vec{E}$ is calculated. The magnitude of the force that acts on it is

$$F = q_0 E$$

$$= \frac{1}{4\pi \epsilon_0} \frac{q q_0}{r^2}$$

$$\therefore F = \frac{1}{4\pi \epsilon_0} \frac{q q_0}{r^2}$$

Which is precisely coulomb's law. Thus we have deduced coulomb's law from Gauss's law.

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21. Write down the Application of Gauss's law.

Application of Gauss's law:

Although Gauss's law is always true it is only a useful tool to calculate the electric field if the charge distribution is symmetric:

1. If the charge distribution has **spherical symmetry**, then Gauss's law can be used with concentric spheres as Gaussian surfaces.
2. If the charge distribution has **cylindrical symmetry**, then Gauss's law can be used with coaxial cylinders as Gaussian surfaces.
3. If the charge distribution has **plane symmetr**y, then Gauss's law can be used with pill boxes as Gaussian surfaces.

22. An infinite length rod have linear charge density ρ. Calculate E for the rod at a distance 'r' from the rod.

Or, Find an expression of $\vec{E}$ for a rod (sylindrical surface).

Let us consider a section of an infinite rod with uniformly charge density ρ, is constant for all points on the line.

Let the rod be surrounded by a cylindrical Gaussian surface of radius r and length h.

If $\vec{E}$ is the electric field, according to Gauss's law we can write

$$\epsilon_0 \oint \vec{E} \,.\, dA = \rho h$$

$$dA, \epsilon_0 \oint E \,.\, dA = \rho h$$

$$dA, \epsilon_0 E \oint dA = \rho h$$

$$dA, \epsilon_0 E(2\pi r h) = \rho h$$

$$\therefore E = \frac{\lambda}{2\pi \epsilon_0 r}$$

The direction of $\vec{E}$ is radially outward for a line of positive charge.

In vector form,

$$\vec{E} = \frac{\lambda}{2\pi \epsilon_0 r^2}\hat{r}$$

23. What is electric potential energy?

Electric potential:

The work done in bringing a positive charge from infinity up to a point is called the electric potential at that point.

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If the work done is W and the charge is q_0, the electric potential V, can be written as

$$V = W/q_0$$

24. What do you understand by equipotential surface?

Equipotential surface:

An equipotential surface is a three dimensional surface on which the electric potential is the same at every point.

25. What is meant by potential difference?

The potential difference between two points can be defined as the work done W_{AB} in moving a test charge q_0 from a point A to a point B keeping the charge in equilibrium. Thus,

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

Unit: Volt $= \frac{\text{Joule}}{\text{Coulomb}}$

27. Derive an expression for potential due to point

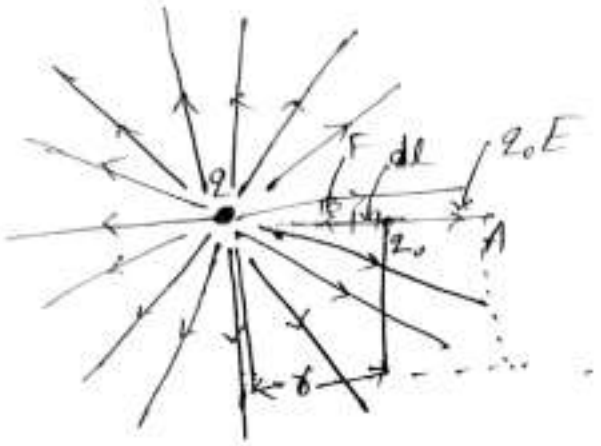


Fig: A test charge Q_0 is moved A to B in field set up by a point charge Q.

Let us consider A, B and Q_0 lie on a straight

Let us compute the potential difference betwee

points A and B, assuming that a test cha

Q_0 is moved without acceleration, along a radi

line from A to B.

$\vec{E}$ point to the right and $d\vec{l}$ to the left

Therefore, $\vec{E} \cdot d\vec{l} = E\,dl \cos 180°$

$= -E\,dl$

As we move a distance dl to the left, we

moving in the direction of decreasing r.

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