Chapter two

Electric current, simple circuits and electric measurement

1. What is electric current?

Answer:

The amount of charge that flow's through a particular cross-sectional area of conductor per unit time is called electric current.

Its unit in Ampere (A) e. $1\ A = 1\ cs^{-1}$.

2. What do you understand by current? Find an expression for current.

Answer:

Current: Flow of charge per unit time is called current.

$$I = \frac{Q}{t}$$

Let us consider,

n = charge per unit volume

Q = value of one charge

v = velocity of charge

Δx = distance

A = Cross section area

I = current

t = time

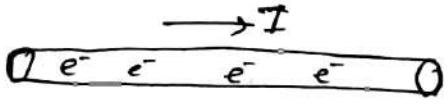


Figure: conductivity wire.

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Volume of the section is $= A\Delta x$

Total charge on that section : $n(A\Delta x)$

So, Amount of charge, $Q = nA\Delta x \times q$

Again, ~~volum~~ velocity, $V = \frac{\Delta x}{t}$

$\Delta x = vt$

$\therefore$ Current, $I = \frac{Q}{t}$

$= \frac{nA\Delta x \times q}{t}$

$= \frac{nAvt \times q}{t}$

$$\boxed{I = nAvq}$$

3. What do understand by current density? Find an expression for current density.

Current density:

Current density: Current per unit Area is called density.

$I = nAvq$

$\therefore J = \frac{I}{A} = \frac{nAvq}{A} = nvq$

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4. What is Resistance?

Resistance: The property of an object by which flow of charges are ~~obstacled~~ obstacled is called resistance.

5. What is conductance?

Conductance:

The property of an object by which flow charges are assisted is called conductance.

6. State and explain ohm's Law?

Ohm's Law:

Statement: The ratio of potential difference (V) between any two points on a conductor to the current I flowing between them is constant, provided the temperature of the conductor does not change.

Explain: According to the statements

$$I \propto V$$

$$\Rightarrow I = \text{constant} \times V$$

$$\Rightarrow I = G V$$

$$\Rightarrow I = \frac{1}{R} V$$

$$\therefore \boxed{V = IR}$$

Here,
G = conductance
R = Resistance

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7. Find out the expression energy/power (P) in an electric circuit.

Energy trans-ferred, du

$$du = V_{ab}\, dq$$

$$\Rightarrow du = i\,dt\, V_{ab} \quad \Big| \; dq = i\,dt$$

$$\Rightarrow \frac{du}{dt} = i\, V_{ab}$$

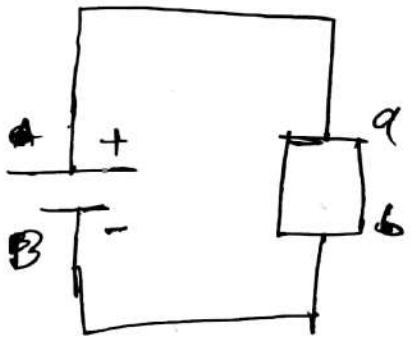


Fig: A battery B sets up a current in a circuit containing a "black box"

Work done per unit time is power P.

$$\therefore P = Vi$$

$$\Rightarrow P = i^2 R \quad [V = iR]$$

$$\Rightarrow P = V \cdot \frac{V}{R} \quad I = \frac{V}{R}$$

$$\therefore \boxed{P = \frac{V^2}{R}}$$

8. What do you understand by Resistivity?

Resistivity: Resistivity is the property of an object. The symbol of resistivity is ρ. It is defined

$$\rho = \frac{E}{j}$$

$$\therefore \rho = \frac{\frac{V}{l}}{\frac{i}{A}}$$

$$= \frac{V}{i} \frac{A}{l} = R \times \frac{A}{l}$$

We know,

$$V = El$$

$$\Rightarrow E = \frac{V}{l}$$

$$j = \frac{i}{A}$$

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Q. State and explain kirchhoff's Law.

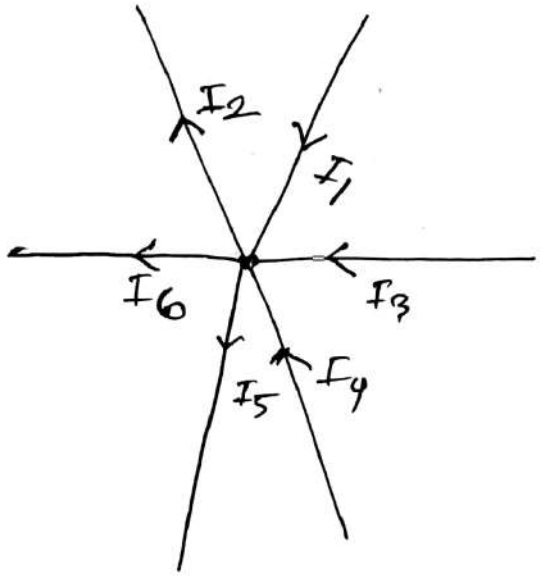
First Law:

In a junction of an electric circuit, the algebraic sum of currents is zero.

That is $\sum I = 0$.

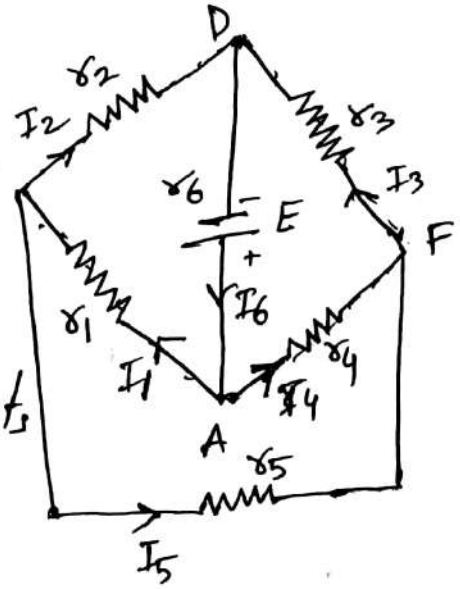
According to kirchhoff's Law

$$I_1 + I_3 + I_4 - I_2 - I_5 - I_6 = 0$$



2nd Law:

In a closed circuit, the algebraic sum of the products of resistance of different parts and their corresponding currents is equal to the total electromotive force in the circuit.



i.e. $\sum I\gamma = \sum E$

In ABDA closed path,

$$I_1\gamma_1 + I_2\gamma_2 + I_6\gamma_6 = E .$$

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10. Calculate Equivalent resistance for Series and parallel connection circuit.

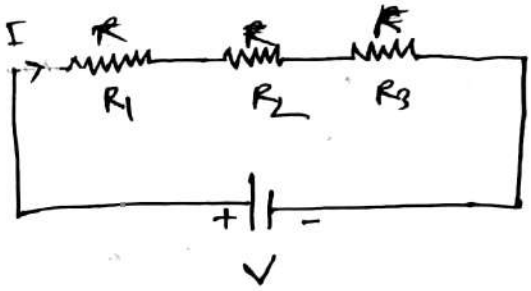
Equivalent resistance for series connection circuit:

Here,

$$V = V_1 + V_2 + V_3$$

$$IR_s = IR_1 + IR_2 + IR_3$$

$$\therefore R_s = R_1 + R_2 + R_3$$



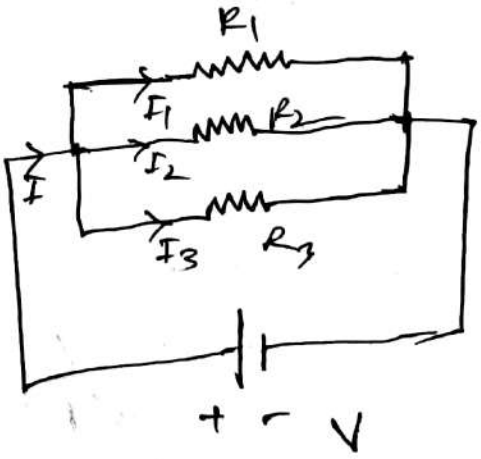
Equivalent resistance for parallel connection circuit:

Here,

$$I = I_1 + I_2 + I_3$$

$$\Rightarrow \frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\therefore \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



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11. Define : Electromotive force.

The work done in bringing a unit charge, starting from any point of the electric circuit and traversing the entire circuit including the cell and back to the same point i.e the electric energy supplied by the cell is called the electromotive force of the cell.

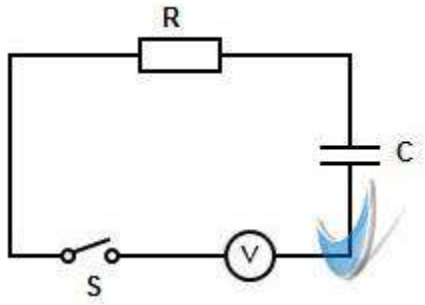
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12. For an RC circuit, when the switch is connected to the battery. Find the expression for charge q and current I. Or, Prove $q = q_0(1 - e^{-\frac{t}{Rc}})$ Or, $q = CV(1 - e^{-\frac{t}{Rc}})$

Capacitor charging Equation Derivation:

Here derives the expression to obtain the instantaneous voltage across a charging capacitor as a function of time,that is V (t). Consider a capacitor connected in series with a resistor, to a constant DC supply through a switch S. 'C' is the value of capacitance and 'R' is the resistance value. The 'V' is the Voltage of the DC source and 'v'is the instantaneous voltage across the capacitor.

When the switch 'S' is closed, the current flows through the capacitor and it charges towards the voltage V from value 0. As the capacitor charges, the voltage across the capacitor increases and the current through the circuit gradually decrease. For an uncharged capacitor, the current through the circuit will be maximum at the instant of switching. And the charging currents reaches approximately equal to zero as the potential across the capacitor becomes equal to the Source voltage 'V'.



Considering voltage law, the source voltage will be equal to the total voltage drop of the circuit. Therefore,

$$V = iR + v \quad \textit{i-current}$$
$$V - v = iR$$
$$i = \frac{dQ}{dt}$$
$$Q = Cv \quad \textit{Q-charge}$$
$$i = C\frac{dv}{dt}, \text{ take } C \text{ outside the derivative as it is a constant value}$$
$$V - v = R \cdot C\frac{dv}{dt}$$

Rearrange the equation to perform the integration function,

$$\frac{dt}{RC} = \frac{dv}{V-v}$$

Intergrate both sides,

$$\int \frac{dt}{RC} = \int \frac{dv}{V-v}$$

RHS simplification,

$$
\begin{aligned}
&u = V - v \\
&\tfrac{du}{dv} = -1 \\
&du = -dv \\
&-\int \tfrac{1}{u} du = -logu \\
&-logu = -log(V - v)
\end{aligned}
$$

On integrating we get,

$$
\begin{aligned}
&\tfrac{t}{RC} + k = -log(V - v) \\
&-\tfrac{t}{RC} + k = log(V - v)
\end{aligned}
$$

As we are considering an uncharged capacitor (zero initial voltage), the value of constant 'K ' can be obtained by substituting the initial conditions of the time and voltage. At the instant of closing the switch, the initial condition of time is t=0 and voltage across the capacitor is v=0.

Thus we get, logV=k for t=0 and v=0.

Substitute the value of K as $logV$,

$$
\begin{aligned}
&-\tfrac{t}{RC} + logV = log(V - v) \\
&-\tfrac{t}{RC} = log(V - v) - logV \\
&-\tfrac{t}{RC} = log\tfrac{(V-v)}{V}
\end{aligned}
$$

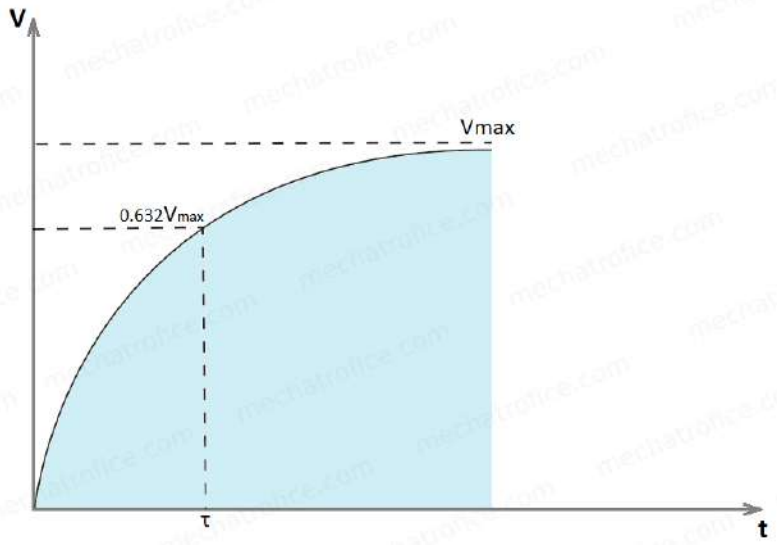
Taking exponential on both sides,

$$
\begin{aligned}
&e^{\frac{-t}{RC}} = \tfrac{V-v}{V} \\
&Ve^{\frac{-t}{RC}} = V - v \\
&v = V - Ve^{\frac{-t}{RC}} \\
&v = V(1 - e^{\frac{-t}{RC}})
\end{aligned}
$$

From the above expression, it is clear that the instantaneous voltage will be a result of factors such as capacitance, resistance in series with the capacitor, time and the applied voltage value.

As the value of the constant RC increases, the value of exponential function also increases. That is the rate of voltage rise across the capacitor will be lesser with respect to time. That shows the charging time of the capacitor increase with the increase in the time constant RC.

As the value of time 't' increases, the term reduces and it means the voltage across the capacitor is nearly reaching its saturation value.



Charge q and charging current i of a capacitor

The expression for the voltage across a charging capacitor is derived as,

$\mathbf{v = V(1- e^{-t/RC})}$ → equation (1).

V – source voltage
v – instantaneous voltage
C– capacitance
R – resistance
t– time
The voltage of a charged capacitor, **V = Q/C**.
Q– Maximum charge
The instantaneous voltage, **v = q/C**.
q– instantaneous charge
$\mathbf{q/C = Q/C\ (1- e^{-t/RC})}$

$\mathbf{q = Q\ (1- e^{-t/RC})}$

Charging current

For a capacitor, the flow of the charging current decreases gradually to zero in an exponential decay function with respect to time.

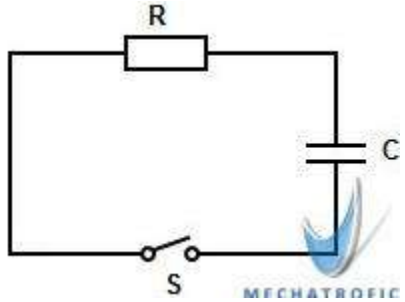
From the voltage law,

ν = V(1- e $^{-t/RC}$)
ν = V – V e $^{-t/RC}$
V – ν = V e $^{-t/RC}$ →equation(2)
The source voltage, V = voltage drop across the resistor (IR) + voltage across the capacitor (**ν**).
V = i R + ν
V – ν = i R
Substitute **V – ν = i R** in the equation 2.
Therefore, **i R = V e $^{-t/RC}$**
i = (V /R) e $^{-t/RC}$

As V is the source voltage and R is the resistance, **V/R** will be the maximum value of current that can flow through the circuit.
V/R =Imax
i = Imax e $^{-t/RC}$

Capacitor Discharge Equation Derivation

For a discharging capacitor, the voltage across the capacitor *v* discharges towards 0.



Applying Kirchhoff's voltage law, *v* is equal to the voltage drop across the resistor R.

$$v + iR = 0 \quad \text{——} ①$$

$$i = \frac{dQ}{dt} = C\frac{dv}{dt}$$

The current i through the resistor is rewritten as above and substituted in equation 1.

$$V + RC\frac{dv}{dt} = 0$$

$$V = -RC\frac{dv}{dt}$$

$$-\frac{dt}{RC} = \frac{dv}{V}$$

By integrating and rearranging the above equation we get,

$$-\frac{1}{RC}\int dt = \int \frac{1}{V}dv$$

$$-\frac{t}{RC} + k = \log V$$

$$logv = \frac{-t}{RC} + \log V$$

$$logv - \log V = -\frac{t}{RC}$$

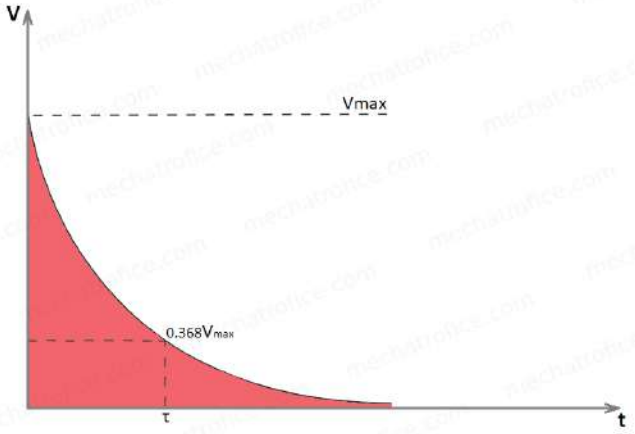
$$log\frac{v}{V} = \frac{-t}{RC}$$

Applying exponential function,

$$\frac{v}{V} = e^{-t/RC}$$

$$v = Ve^{-t/RC}$$

The instantaneous voltage across a discharging capacitor is v = V e -t/RC
Instantaneous charge, q = Q e -t/RC
Instantaneous current, i = – Imax e -t/RC



From the above equations, it is clear that the voltage, current, and charge of a capacitor decay exponentially during the discharge. The discharge current has a negative sign because its direction is opposite to the charging current.

Deduction of Wheatstone Bridge Principle to measure Resistance

Definition : If four resistances are connected in series in such a way that connecting the first end of the first resistance and last end of the last resistance forms a closed circuit and if one terminal of a cell is connected to a junction of any two resistances and the other terminal to the junction of other two resistances and if a galvanometer is connected to other two junctions, then this circuit is called Wheatstone bridge.

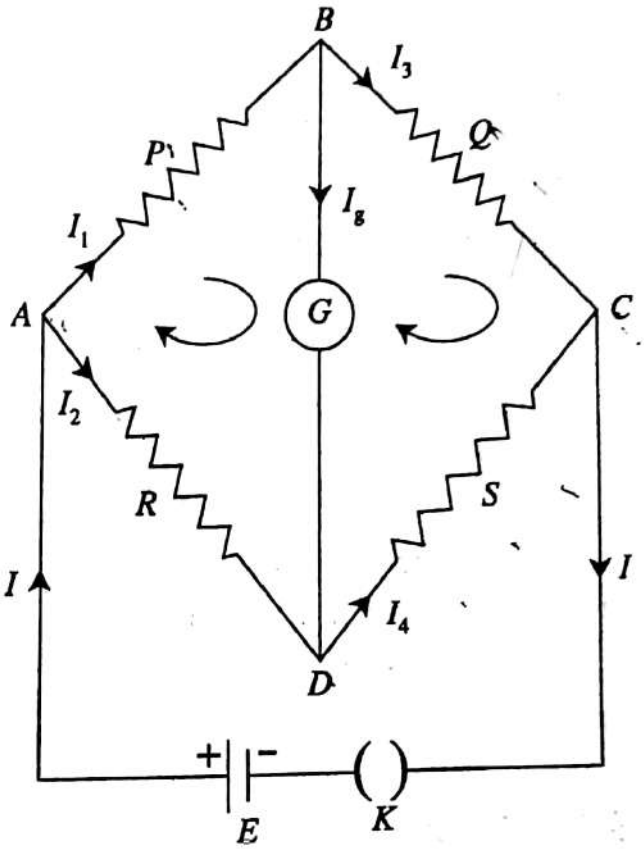


Fig 3.10

In fig : 3.10, four resistances P, Q, S and R are arranged one after another to form a closed circuit.

When key is closed, current I emerging from the cell E comes to point A and is divided into two parts I_1 and I_2. These two current I_1 and I_2 flowing through P and R respectively reach B and D. Now, if potential of B is greater than that of D, then a portion of I_1, namely I_g passing through galvanometer meets I_2 at D. This combined current I_4 passes through S to reach C. The rest portion of I_1 i.e., I_3 passing through Q and reaching at C meets I_4 and becomes the total current I and returns to E. On the other hand, if potential of D is greater than that of B, then a portion of I_2 passing through galvanometer meets I_1 at B and passes through Q to reach C. The rest portion of I_2 passing through S and reaching at C becomes the total current I and returns to E.

But, If the potentials of points B and D are equal, then no current will flow through the galvanometer *i.e.*, $I_g = 0$ and the pointer of the galvanometer will not deflect. This is called ***the equilibrium position*** **or** ***null position*** of the Wheatstone bridge.

Applying Kirchhoff's first law at point B, we get

$$I_1 - I_g - I_3 = 0$$

$$\text{or, } I_1 = I_g + I_3 \quad \ldots \quad \ldots \quad (3.39)$$

Again, applying Kirchhoff's first law at point D, we get

$$I_2 + I_g - I_4 = 0$$

$$\text{or, } I_2 + I_g = I_4 \quad \ldots \quad \ldots \quad \ldots \quad (3.40)$$

Now, applying Kirchhoff's second law in $ABDA$ closed circuit, we get

$$I_1P + I_gG - I_2R = 0 \quad \ldots \quad \ldots \quad \ldots \quad (3.41)$$

Again, applying Kirchhoff's second law in $BCDB$ closed circuit, we get

$$I_3Q - I_4S - I_gG = 0 \quad \ldots \quad \ldots \quad \ldots \quad (3.42)$$

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When no current flows through galvanometer *i.e.*, at the equilibrium position of Wheatstone bridge, $I_g = 0$. Then from eq. (3.39) we get, $I_1 = I_3$

From eq. (3.40), we get, $I_2 = I_4$

and from eq. (3.41) we get,

$$I_1P - I_2R = 0 \text{ or, } \frac{I_2}{I_1} = \frac{P}{R} \quad \ldots \quad \ldots \quad \ldots \quad (3.43)$$

Now, from eq. (3.42) we get,

$$I_3Q - I_4S = 0 \text{ or, } \frac{I_4}{I_3} = \frac{Q}{S}$$

$$\because I_1 = I_3 \text{ and } I_2 = I_4 \qquad \therefore \frac{I_2}{I_1} = \frac{Q}{S} \quad \ldots \quad \ldots \quad (3.44)$$

Now from equations (3.43) and (3.44), we get

$$\frac{P}{R} = \frac{Q}{S} \qquad \therefore \frac{P}{Q} = \frac{R}{S} \quad \ldots \quad \ldots \quad \ldots \quad (3.45)$$

This is the condition of equilibrium of Wheatstone bridge. Equation (3.45) Shows that among P, Q, R and S, if any three resistances are known then fourth resistance could be found.

The four resistances P, Q, R and S of the Wheatstone bridge are known as the first arm second arm, third arm and fourth arm respectively.

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Mathematical problems

Chapter two

1. An aluminum wire whose diameter is 0.10 in. is welded end to end to a copper wire with a diameter of 0.064 in. The composite wire carries a steady current of 10 amp. What is the current density in each wire?

Answer:

For Aluminum,

$$j_{Al} = \frac{i}{A}$$

$$= \frac{10}{0.0079 \; 7.854 \times 10^{-3}}$$

$$= 1273.24 \text{ amp/in}^2$$

Here, [Al]

Current, $i = 10$ amp

diameter, $d = 0.10$ in

$\therefore$ radius, $r = \frac{d}{2}$

$= \frac{0.10}{2}$ in

$= 0.05$ in

Area, $A = \pi r^2$

$= 3.1416 \times (0.05)^2$

$= 7.854 \times 10^{-3}$ in^2

$= 0.0079$ in^2

For copper,

$$j_{cu} = \frac{i}{A}$$

$$= \frac{10}{0.0032}$$

$$= 3105.59 \text{ amp/in}^2$$

[cu]

$d = 0.064$ in

$r = \frac{d}{2}$

$= \frac{0.064}{2}$ in

$= 0.032$ in

$A = \pi r^2$

$= \pi \times (0.032)^2$ in^2

$= 0.0032$ in^2

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2. A rectangular carbon block has dimensions 1.0 cm × 1.0 cm × 50 cm. (a) What is the resistance measured between the two square ends? (b) Between two opposing rectangular faces? The resistivity of carbon at 20°C is 3.5×10^{-5} ohm-m.

Answer :

Here,

Resistivity, $\rho = 3.5 \times 10^{-5}$ ohm-m

The area of square end is, $A = 1.0\ cm \times 1.0\ cm$

$\Rightarrow A = 1.0\ cm^2$

$= 1.0 \times 10^{-4}\ m^2$

Length, $l = 50\ cm$

$= 0.50\ m$

Again,

The area of the rectangular faces

$A = 1.0\ cm \times 50\ cm$

$= 50\ cm^2$

$= 50 \times 10^{-4}\ m^2$

$= 5 \times 10^{-3}\ m^2$

and length, $l = 1\ cm$

$\Rightarrow l = 1 \times 10^{-2}\ m$

(a) The resistance between two square ends.

$$R = \frac{\rho l}{A}$$

$$= \frac{3.5 \times 10^{-5} \times 0.50}{1.0 \times 10^{-4}}$$

$$= 0.175\ \text{ohm}$$

(b) The resistance between opposing rectangular faces.

$$R = \frac{\rho l}{A}$$

$$= \frac{3.5 \times 10^{-5} \times 1 \times 10^{-2}}{5 \times 10^{-3}}$$

$$= 7 \times 10^{-5}\ \text{ohm}$$

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